Charles Hutton: scientist, mathematician and the density of the Earth

Robin Johnson
Outline

- his life
- his science
- his mathematics & teaching
- the Schiehallion experiment
- overview & summary
Charles Hutton: his life

Born on Percy Street; father Henry, mother Eleanor.

Taught to read (Percy St.), and then to write (Benwell).

Educated at Rev. Mr. Ivison’s school in Jesmond
   – and he then took over the school.

He moved the school to Stotes Hall, with three further moves as the school expanded.
Teaching by day, studying mathematics in the evenings at Mr. James’ Mathematical School in Newcastle.

Started teaching at the school; married Isabella in 1760; took over the school (advert).

In 1773, he was appointed Professor of Mathematics at the Royal Military Academy in Woolwich.

Left Isabella and his children in Newcastle.

Elected to the Royal Society in 1774, and was its foreign secretary 1779-’83.
Isabella died in 1785; he remarried (Margaret).

Four daughters by first wife and one by his second, but two predeceased him.

In 1786 he began to suffer from pulmonary disorders, so decided to move away from the Thames.

New Royal Military Academy – closed in 1939

by Andrew Morton (1802–1845)
Ill health forced him to resign his Chair in 1807; he died in 1823.

In his last year, a fund was set up to honour him: to cover the cost of a marble bust and, so much was raised, also a medal.
Charles Hutton: his science

1770: was asked to produce a survey of Newcastle and its environs.

1772: wrote ‘The Principles of Bridges’.

Appointed Professor at RMA in 1773.

Elected to Royal Society in 1774.
1776: *A new and general method for finding simple and quickly converging series,*

1778: *The force of fired gunpowder and the velocity of cannon balls* — *for which he received the Copley Medal.*

As asked by the Royal Society to perform calculations to determine the mass and density of the Earth;

published in 1778 (and reprinted in 1812 in a collection of his works).
1781: set of *Mathematical Tables* for the Board of Longitude.

1795: *Mathematical and Philosophical Dictionary*.

1803: *Recreations in Mathematical and natural Philosophy*.

Abridged the *Philosophical Transactions*; finally published in 1809.
First text: 1764: *The Schoolmasters Guide, or a complete system of practical arithmetic* (dedicated to Robert Shafto)

Saw an opening to help & educate schoolmasters, so in 1766 and 1767 he advertised:

‘Any schoolmaster, in town or country, who is desirous of improvement in any branch of the mathematics, by applying to Mr Hutton, may be instructed.’
1770: *Treatise on mensuration both in theory and practice*

which included illustrations that were the first assignment of Thomas Bewick.

1781: *Tables of the Products and Powers of Numbers.*

1785: *Mathematical Tables,* based on the work produced for the Board of Longitude.
and for the use mainly at the Military Academy:

1784: *The Compendious Measurer*

1787: *Elements of Conic Sections*

1798: *Course of Mathematics for cadets of the Royal Military Academy.*
Other interests

From 1764 contributed to *The Ladies’ Diary* and was its editor from 1773-1817.

He started his own small periodical (*Miscellane Mathematica*) and published five volumes of *The Diarian Miscellany.*
The Schiehallion experiment

The aim was to find the density of the Earth (and then for other celestial bodies).

Basic idea: use deviations of a pendulum hanging close to a large mass.

This had been rejected as unrealistic by Newton!

BUT a proposal was put to the Royal Society (by Nevil Maskelyne) in 1772

– and the Committee of Attraction was formed.
Initial stage: first find your mountain.....

This fell to Charles Mason who, in 1773, proposed Schiehallion as the best, reasonably accessible mountain.

But Mason was not prepared to undertake the task involved in the measurements....

so Maskelyne organised it; it was in two parts:

measure the deviation of the pendulum and find the volume (and then mass) of the mountain.
First part – the most difficult – was completed in 1774.

This required the measurement of the deviation of the pendulum, relative to the stars.

Two observatories built, on North and South flanks of the mountain.

Taken into account:

curvature of the Earth, its precession, its nutation and the aberration of the light from the stars.
Results published in 1775.

Maskelyne gave a rough estimate of the volume, and then the mass, of the mountain.....

and on this basis, the average deviation of the pendulum would be 21 secs of an arc

BUT he measured about 11 ½ secs of an arc

.....so he estimated that the density of the Earth was approximately twice that of the mountain.

Maskelyne was awarded the 1775 Copley Medal for this work.
Second stage: produce an accurate survey of the mountain.

Done by a team of surveyors – but bad weather meant that this was not completed until 1776.

Hutton not involved in this – he was simply asked to perform the calculations….

so could give no guidance as to where to take measurements, and how many were needed

(data often taken as convenient to the surveyors on the mountain).
The data points:
Method used by Hutton:

- divide the mountain into many prisms
- find their volumes and then masses
- find the gravitational attraction to the pendulum
- add all contributions

But he found that a number of the prisms did not have suitable (or had insufficient) data points; in particular, some heights were missing.....

WHAT TO DO?
Could not ignore them – some of the mountain would be missing!

He hit on the idea of interpolating: he imagined lines drawn horizontally at the same height from other data points nearby….

...he had introduce contour lines!

Calculations took until 1778, when he published all his results.
His results

He took the density of the rock to be $2,500 \text{ kg/m}^3$ uniform throughout the mountain, and so determined a value for the mass of Schiehallion; he then used the deflection results of Maskelyne, to estimate the mean density of the Earth as about $4,500 \text{ kg/m}^3$.

The current accepted figure is about $5,515 \text{ kg/m}^3$.

Comment: With all the errors and assumptions involved, a quite creditable result for the time.
To complete the scientific story….

the essential idea behind this experiment was repeated by Henry Cavendish in 1798; he used a torsion balance to measure the attraction between two large lead spheres….. and produced a result within 1% of the current value.
....and the final word

calculations have been done using Maskelyne’s deflection data

with modern GPS techniques to produce a digital-elevation model of Schiehallion

and a more comprehensive understanding of the geology of the mountain i.e. its varying density

to give a mean density of the Earth

\[ 5,480 \pm 250 \text{ kg/m}^3. \]
Overview and Summary

We have described

- Charles Hutton – his life & his teaching successes
- his scientific exploits in general
- his mathematical & other interests
- the Schiehallion experiment & his use of contours
The End